RELATIVISTIC STUDIES OF CLOSE NEUTRON STAR BINARIES

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We discuss (3+1) dimensional general relativistic hydrodynamic simulations of close neutron star binary systems. The relativistic field equations are solved at each time slice with a spatial 3-metric chosen to be conformally flat. Against this solution the hydrodynamic variables and gravitational radiation are allowed to respond. We have studied four physical processes which occur as the stars approach merger. These include: 1) the relaxation to a hydrodynamic state of almost no spin; 2) relativistically driven compression, heating, and neutrino emission; 3) collapse to two black holes; and 4) orbit inspiral occurring at a lower frequency than previously expected. We give a brief account of the physical origin of these effects and an explanation of why they do not appear in models based upon, 1PN hydrodynamics, a weak field multipole expansion, a tidal analysis, or a rigidly corotating velocity field. The implication of these results for gravity wave detectors is also discussed.

The physical processes occurring during the last orbits of a neutron star binary are currently a subject of intense interest.[1−10] In part, the recent surge in interest stems from relativistic numerical hydrodynamic simulations in which it has been noted [1,2,3] that as the stars approach their final orbits they experience compression. Indeed, for an appropriate equation of state, numerical simulations [3] indicate that binary neutron stars heat significantly before individually collapsing toward black holes many seconds prior to merger. The orbit frequency is also significantly lower than that of Newtonian or post-Newtonian point sources, and the inner most stable orbit occurs at a larger separation distance.[2] All of these effects could have a significant impact on the anticipated gravity wave signal from merging neutron stars. They could also provide an energy source for cosmological gamma-ray bursts.[3]

However, a number of recent papers [4−10] have not observed this effect in Newtonian,[4] 1PN,[5,6] weak field multipole expansions,[7,8,9] or in binaries in which rigid corotation has been imposed.[10] Moreover, this flurry of activity has caused some confusion as to the physics responsible for the effects observed in the numerical calculations. Here, we present a brief derivation of the physics which drives the compression and discuss how such terms are missed in the various approximation schemes. We describe simulations which demonstrate that the compression forces do not appear in simple linear motion or rigid corotation. We also summarize the implications of these results on the gravity wave signal of close neutron star binaries.

The basic physical processes which induce compression can be traced to completely general terms in the hydrodynamic equations of motion.[3] We begin with the usual ADM (3+1) metric [12,13] in which there is a slicing of the spacetime into a

one-parameter family of three-dimensional hypersurfaces γ_{ij} separated by differential displacements in a time-like coordinate α,

$$ds^2 = -(\alpha^2 - \beta_i \beta^i)dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j \quad . \tag{1}$$

The conformally flat metric condition (*CFC*) expresses the three metric of Eq. (1) as $\gamma_{ij} = \phi^4 \delta_{ij}$. It is common practice to impose this condition when solving the initial value problem in numerical relativity (which is in essence what we do). One question, however, is the amount of hidden radiation [14] contained in the CFC solution. We have estimated this by decomposing the extrinsic curvature into longitudinal K_L^{ij} and transverse K_T^{ij} components as proposed by York.[15] By this order-of-magnitude estimate, we find that the "hidden" gravitational radiation energy density is a small fraction of the total gravitational mass energy of the system, $\int K_T^{ij} K_{Tij} \frac{dV}{8\pi} \approx 2 \times 10^{-5}$ $\mathrm{M_G}$. Similarly, the multipole estimate of the power loss in gravitational radiation is a small fraction of the energy in orbital motion $\dot{J}/\omega J \sim 10^{-4}$. Hence, the CFC is probably a good approximation to the initial data for the binaries we study.

The vanishing of the spatial components of the divergence of the energy momentum tensor $(T_\mu{}^i)_{;\mu} = 0$ leads to an evolution equation for the covariant four momentum,

$$\begin{aligned} \dot{S}_i \quad &+ \quad S_i \frac{\dot{\gamma}}{\gamma} - \frac{1}{\gamma} \frac{\partial}{\partial x^j}(S_i V^j \gamma) + \frac{\alpha \partial P}{\partial x^i} - S_j \frac{\partial \beta^j}{\partial x^i} \\ &+ \quad (\rho(1+\epsilon) + P)\left(W^2 \frac{\partial \alpha}{\partial x^i} + \alpha \frac{U_j U_k}{2} \frac{\partial \gamma^{jk}}{\partial x^i} \right) = 0 \quad . \end{aligned} \tag{2}$$

In an orbiting system it is convenient to allow β^j to follow the the orbital motion of the stars. In which case, the term $S_j(\partial \beta^j / \partial x^i)$ contains the centrifugal force (plus some small frame drag). The term containing $\partial \alpha / \partial x^i$ is the analog of the Newtonian gravitational force.

The term with $(U_j U_k/2)\partial \gamma^{jk}/\partial x^i$ is the compression driving force. It does not have a Newtonian analog. This term vanishes for a star at rest with respect to the observer or in the flat-space limit. However, for a star with fluid motion in curved space, it describes an additional force represented as a product of velocities times the gradient of the three metric. For simple linear motion the effects of this term should cancel to leave the stellar structure unchanged. Similarly, this term appears to cancel [10] for fluid motion in which the four velocity can be taken as proportional to a simple Killing vector. However, for more general states of motion (e.g. noncorotating stars, differential rotation, meridional circulation, turbulent flow, etc.) the effects of this force must be evaluated numerically. Indeed, the sign of this force is such that a lower energy configuration for a binary star than that of rigid corotation is obtained by allowing the fluid to respond to this force term. We find [16] that the numerical relaxation of binary stars from corotation (or any other initial spin configuration) produces a state of almost no net spin in which the central density and gravitational binding energy increase.

We have performed numerous numerical tests which substantiate that this term does indeed vanish when the hydrodynamics is artificially constrained to uniform

translation, stationary stars in a tidal field, or approximate rigid corotation. Indeed, the constrained stars remain near the central density of an isolated star, while the binary stars show an increase in central density which grows as $\sim (v/c)^4$. We have also analyzed [16] the nature and formation of this state in more detail by imposing an initial rigid angular velocity in the frame of the orbiting stars in the range $-900 < \omega_S < 900$ rad sec^{-1}, corresponding to $-0.03 < J_S/m_0^2 < 0.12$. In each case, the stars relax to a state of almost no net spin within about three sound crossing times (~ 0.6 msec).

We have also used a multipole expansion [2] to extract the gravity wave signal. A striking feature of these simulations is that the frequency varies slowly as the orbit decays (less chirp). This imples a higher signal to noise for gravity wave detectors at low frequencies (~ 100 Hz). At least part of the differences with PN estimates [17] can be attributed to the effects of finite stellar size.[5] However, we attribute most of this difference to time dilation and length contraction in the strong field of the binary.[2] As the stars collapse, an abrupt change in the gravity wave frequency might also be detected. Furthermore, the orbit instability can occur when the specific orbital angular momentum is in excess of unity.[2] Hence, a very large amount of gravity wave emission may accompany the final merger to a single Kerr black hole.

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